Partial Solution Set, Leon §7.4

7.4.1 Determine $\|\cdot\|_F$, $\|\cdot\|_\infty$, and $\|\cdot\|_1$ for each of the following matrices.

(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The norms are $\|A\|_F = \sqrt{2}$, $\|A\|_\infty = \|A\|_1 = 1$.

(d) $= \begin{bmatrix} 0 & 5 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

The norms are $\|A\|_F = 7$, $\|A\|_\infty = 6$, and $\|A\|_1 = 10$.

7.3.3 Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Show that $\|A\|_2 = 1$.

Proof: If $\mathbf{x} = \begin{bmatrix} x_1 \\ x^2 \end{bmatrix} \in \mathbf{R}^2$, then $A\mathbf{x} = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$, so

$$\begin{aligned} \|A\|_2 &= \max_{\mathbf{x} \neq \mathbf{0}} \frac{\|A\mathbf{x}\|_2}{\|\mathbf{x}\|_2} \\ &= \max_{\mathbf{x} \neq \mathbf{0}} \frac{|x_1|}{\sqrt{x_1^2 + x_2^2}} \\ &\leq 1, \end{aligned}$$

with equality when $x_2 = 0$. □

7.4.17 Let $A = \begin{bmatrix} 1 & -0.99 \\ -1 & 1 \end{bmatrix}$. Find A^{-1} and $\text{cond}_\infty(A)$.

Solution: $A^{-1} = \begin{bmatrix} 100 & 99 \\ -100 & 100 \end{bmatrix}$, so $\|A\|_\infty = 2$ and $\|A^{-1}\|_\infty = 200$. It follows that $\text{cond}_\infty(A) = 400$

7.4.18 Solve the two given systems and compare solutions. Are the coefficient matrices well-conditioned? Ill-conditioned? Explain.

$$\begin{aligned} 1.0x_1 + 2.0x_2 &= 1.12 \qquad & 1.000x_1 + 2.011x_2 &= 1.120 \\ 2.0x_1 + 3.9x_2 &= 2.16 \qquad & 2.000x_1 + 3.982x_2 &= 2.160 \end{aligned}$$

Solution: The system on the left has solution $x_1 = -0.48$, $x_2 = 0.8$, while the system on the right has solution $x_1 = -2.902$, $x_2 = 2$. The coefficient matrix is ill-conditioned.

MA/Ra, December 8, 2004